Reverse Super Edge Magic Strength of Banana Tree

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Abstract

A graph G is said to be reverse super edge-magic if there exists a bijection $f: V\cup E \rightarrow \{1, 2, ..., p+q\}$ such that $f(uv) - [f(u) + f(v)]$ is a constant, for all $uv \in E$ and $f(V) = \{1, 2, ..., p\}$. Such a bijection is called a reverse super edge-magic labeling and the minimum of all constants is called a reverse super edge-magic strength of the graph G, where the minimum is taken over all reverse super edge-magic labelings of G. In this paper, the reverse super edge-magic labelings and reverse super edge-magic strength of the banana tree BT $(n_1, n_2, ..., n_k)$ are obtained

1 Introduction

The graphs considered here are only finite, simple, undirected graphs. The basic notations and terminologies are as in [3]. In particular, q is the number of edges in G.

Kotzig and Rosa [4] introduced the concept of magic valuation. Ringel and Llado′ [8] called this type of valuation as edge-magic labeling.Further Enomoto et al [2] restricted the notion of edge-magic labeling of a graph to obtain the definition of super edge-magic labeling. An edge- magic labeling of a graph G is called a super edge-magic labeling of G if $f(V) = \{1, 2, \ldots, p\}$ and $f(E) = \{p+1, p+2, \ldots, p+q\}$. A graph G is super edge-magic if it has super edge-magic labeling. The concept of a super edge–magic strength of a super edge-magic graph is introduced by Avadayappan et al [1]. They established the super edge-magic strength of the path P_{2n} of odd length, P_{2n+1} of even length, the star $K_{1, m}$, the bistar $B_{n, n}$, the tree $< K_{1, n} : 2 >$, the graph $(2n+1)P_2$ and the graph $P_n^{(2)}$.

In this paper, new concepts of reverse super edge- magic labeling as well as reverse super edge-magic strength of a graph are introduced. The reverse super edge-magic labeling is a bijection $f: V\cup E \rightarrow\{1, 2, ..., p+q\}$ such that $f(V) = \{1, 2,..., p\}$, $f(E) = \{p+1, p+2, ..., p + q\}$ and $f(uv) - [f(u) + f(v)]$ is a constant, for every edge uv of E. A graph G is said to be reverse super edge-magic if there exists such a bijection f of G.

Again for any reverse super edge-magic labeling f of G, there is a constant $c^{-1}(f)$ such that $f(uv) - [f(u) + f(v)] = c^{-1}(f)$, for any edge $uv\in E$. The reverse super edge-magic strength is denoted by rsems (G) and is defined as the minimum of all $c^{-1}(f)$, where the minimum is taken over all reverse super edge-magic labeling f of G.

Therefore rsems (G) = min $\{c^{-1}(f)$: f is a reverse super edge-magic labeling of G$\}$.

Further the reverse super edge-magic labeling as well as reverse super edge-magic strength of the Banana tree BT $(n_1,n_2,\ldots,n_k)$ are obtained.

To proceed further, the following definations and results are useful.

***Definition* 1.1:** The graph BT$(n_1,n_2,\ldots,n_k)$ is a Banana tree obtained by connecting a new vertex v to one leaf v_{ii} of each of any number of stars K_{1,n_i} with $n_i \geq i$, for $1 \leq i \leq n$, where v is not a vertex of any star.

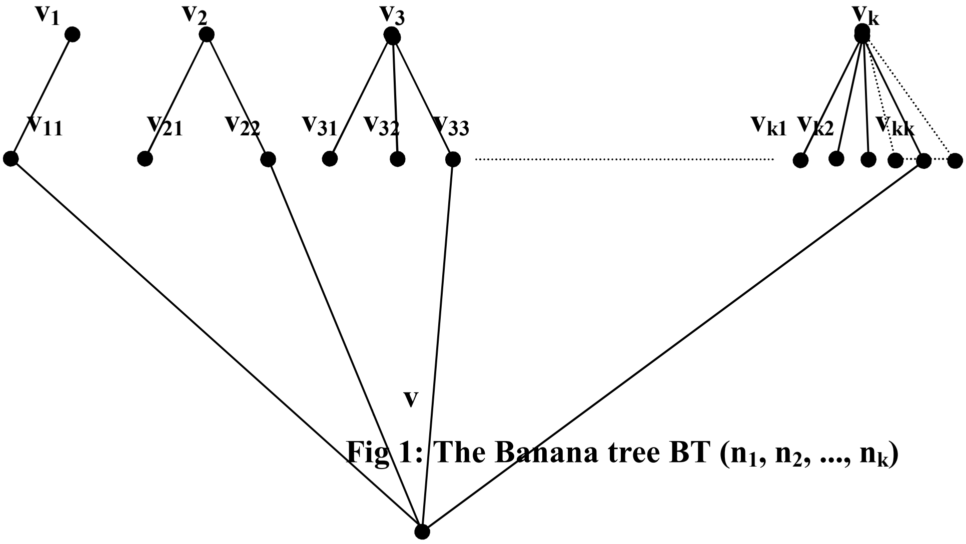

Fig 1: The Banana tree BT $(n_1, n_2, \ldots, n_k)$

***Note* 1.2:** Let f be any arbitrary reverse super edge-magic labeling of a graph G with reverse super edge-magic constant $c^{-1}(f)$. Then by adding all the constants obtained at each edge, we get

$$q\,c^{-1}(f) = \sum_{e \in E} f(e) - \sum_{v \in V} d(v) f(v)$$

2 Main Results

In this paper, the reverse super edge-magic labeling and the reverse super edge-magic strength of the Banana tree BT$(n_1, n_2, \ldots, n_k)$.

Theorem 2.1: rsems (BT $(n_1,n_2,\ldots, n_k)$) = $\sum_{i=1}^{k} n_i - 1$

Proof: Let $v_1, v_2, \ldots, v_k$ be the central vertices of the stars $K_{1,n_1}, K_{1,n_2} .., K_{1,n_k}$ respectively. Let v_{ij} $(1 \leq j \leq n_i)$ be the pendent vertices of K_{1,n_i} $(1 \leq i \leq k)$. Let v be the vertex adjacent to v_{11}, $v_{22} \ldots v_{kk}$. Then the Banana tree BT $(n_1,n_2,\ldots,n_k) \cong G$ is defined as follows:

$V = \{ v_1, v_2, \ldots, v_k, v_{ij}, v : 1 \leq i \leq k,\ 1 \leq j \leq n_i\}$ and

$E = \{ v_i v_{ij}, v\, v_{ii} : 1 \leq i \leq k,\ 1 \leq j \leq n_i\}$

where $n_i \geq i$ (Fig 1).

Consider the function f: $V \cup E \to \{1, 2, 3, \ldots, 2k+1+2\sum_{i=1}^{k} n_i\}$ defined by

$f(v) = 1$

$f(v_i) = i + 1, \qquad 1 \le i \le k$

$f(v_{ij}) = k + 1 + j, \qquad 1 \le j \le n_1$

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$f(v_{2j}) = k + n_1 + 1 + j, \qquad 1 \le j \le n_2$

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$f(v_{kj}) = k + \sum_{i=1}^{k-1} n_i + j + 1, \quad 1 \le j \le n_k$

and let us define edge pairs as follows:

The pair $\{1, k + 2\}$ for vv_{11}, labeled by $k + \sum_{i=1}^{k} n_i + 2$

so that $\{2, k + 2\}$ for v_1v_{11}, labeled by $k + \sum_{i=1}^{k} n_i + 3$,

$\{2, k + 3\}$ for v_1v_{12}, labeled by $k + \sum_{i=1}^{k} n_i + 4$,

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$\{2, k + n_1+1\}$ for $v_1v_{1n_1}$, labeled by $k + 2n_1 + \sum_{i=2}^{k} n_i + 2$,

$\{3, k + n_1 + 2\}$ for $v_2 v_{21}$, labeled by $k + 2n_1 + \sum_{i=2}^{k} n_i + 4$,

$\{1, k + n_1 + 3\}$ for vv_{22}, labeled by $k + 2n_1 + \sum_{i=2}^{k} n_i + 3$,

$\{3, k + n_1 + 3\}$ for v_2v_{22}, labeled by $k + 2n_1 + \sum_{i=2}^{k} n_i + 5$,

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$\{3, k + n_1 + n_2 + 1\}$ for $v_2v_{2n_2}$, labeled by $k + 2n_1 + 2n_2 + \sum_{i=3}^{k} n_i + 3$,

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$\{1, k + \sum_{i=1}^{k} n_i\}$ for $vv_{kk,,}$ labeled by $2k + 2\sum_{i=1}^{k-1} n_i + n_k + 1$

$\{k + 1, k + \sum_{i=1}^{k-1} n_i + 2\}$ for $v_k v_{k1}$, labeled by $2k + 2\sum_{i=1}^{k-1} n_i + n_k + 2$

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$\{(k + 1), k + \sum_{i=1}^{k} n_i\}$ for $v_k v_{kk}$, labeled by $2k + 2\sum_{i=1}^{k} n_i$

$\{(k + 1), k + \sum_{i=1}^{k} n_i + 1\}$ for $v_k v_{kk+1}$, labeled by $2k ++1+ 2\sum_{i=1}^{k} n_i$

Then it can be easily verified that $BT(n_1,n_2,..., n_k)$ is reverse super edge-magic with magic number $c^{-1}(f) \leq \sum_{i=1}^{n} n_i - 1$

Next to obtain the reverse super edge-magic strength, by note 1.2, we have

$$(\sum_{i=1}^{k} n_i + k)\, c^{-1}(f) =$$

$$\{(k+1)+\sum_{i=1}^{k} n_i +1] + [(k+1)+2n_1+\sum_{i=1}^{k} n_i + 2]+.....+[(k+1)+2n_1+2n_2+....+2n_{k-1}+n_k+k] + [\ (k+1) +$$

$$\sum_{i=1}^{k} n_1 + 2 + [(k+1) + 2n_1 + \sum_{i=2}^{k} n_i + 3\ ] + ...+ [(k+1) + 2\sum_{i=1}^{k} n_i + k]\} -$$

$$\{1.k + n_1.2 + n_2.3 + ... + n_k\,(k + 1) + 2\,(k + n_1 + 1) + (k + n_1 + 2) +....+$$

$$2(k + n_1 + n_2 + 1) + (k + n_1 + n_2 + 2) +....+ 2(k + n_1 + n_2 + n_3 + 1) +$$

$$(k + n_1 + n_2 + n_3 + 2) + ...+ (k + \sum_{i=1}^{k} n_i) + 2\,(k + \sum_{i=1}^{k} n_i + 1)\}$$

$= (k + 1)\ k + k\sum_{i=1}^{k} n_i + 1 + (n_1+2) + \ldots + (\sum_{i=1}^{k} n_1 + k) + (k+1)\ \sum_{i=3}^{k} n_i + \sum_{i=3}^{k} n_i + \sum_{i=1}^{k} n_i\ \sum_{i=1}^{k} n_i\ + 2 + n_1$

$+ 3 + (n_1+n_2+4) + .. + (\sum_{i=1}^{k} n_i+k) - \{ k + 2n_1 + 3n_2 + \ldots + (k+1)n_k + 2k.k + 2[n_1 + (n_1 + n_2) + (n_1$

$+ n_2 + n_3) + \ldots.) + 2k +$ $k(\sum_{i=1}^{k} n_i - k) + [n_1+2+n_1+3+ \ldots.+ (n_1+n_2+n_3) + (n_1+ n_2+2) +$

$\ldots. + \sum_{i=1}^{k} n_i]\}$

$\geq k\ \Sigma\ n_i + (\sum_{i=1}^{k} n_i)^2 - \sum_{i=1}^{k} n_i - k.$ $= (k + \sum_{i=1}^{k} n_i)$

$(\sum_{i=1}^{k} n_i - 1)$

$\therefore\ c^{-1}(f) \geq \sum_{i=1}^{k} n_i\ - 1$

Hence rsems (BT $(n_1, n_2, \ldots, n_k)$) $= \sum_{i=1}^{k} n_i - 1$

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