Tripled Fixed Point Theorems on Dislocated Quasi B-Metric Spaces

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Abstract:
In this paper we established some new fixed point theorems in dislocated quasi b-metric spaces. The results herein are extension of some well known fixed point theorems in different metric spaces.

1 . Introduction and Preliminaries

In 2015, Chakkrid and Cholatis[4] coined the concept of dislocated quasi b-metric spaces and proved some fixed point theorems for cyclic contractions in such spaces. Mujeeb Ur Rahman and Muhammad Sarwar[10] studied dislocated quasi b-metric spaces and established fixed point theorems for Kannan contraction and Chetterjea type contraction. Cholatis et al.[5] proved fixed point theorems for cyclic weakly contractions in dislocated quasi b-metric space. Berinde and Borcut[15, 8, 9] have initiated study of tripled fixed point and tripled coincidence points for different nonlinear mappings in complete partially ordered metric spaces and established tripled fixed points theorems. In this paper we have established some tripled fixed point theorems for different contraction mappings in complete dislocated quasi b-metric spaces which extend some well known fixed point theorems existing in the literature.

Definition 1.1 *[4] Let X be a non-empty set. Let the mapping $d: X \times X \to [0, \infty)$ and constant $k \geq 1$ satisfy following conditions:*
1. $d(x, y) = 0 = d(y, x) \Rightarrow x = y, \forall x, y \in X$
2. $d(x, y) \leq k[d(x, z) + d(z, y)], \forall x, y, z \in X.$

Then the pair (X, d) is called dislocated quasi-b −metric space or in short dqb −metric space.

The constant k is called coefficient of (X, d). It is clear that $b-$ metric spaces, quasi-b −metric spaces and b −metric-like spaces are dqb −metric spaces but converse is not true.

Example 1.2 *[10] Let $X = R^+$ and for $p > 1, d: X \times X \to [0, \infty)$ be defined as,*

$$d(x, y) = |x - y|^p + |x|^p, \forall x, y \in X.$$

Then (X, d) is dqb −metric space with $k = 2^p > 1$. But (X, d) is not b −metric space and also not dislocated quasi metric space.

Example 1.3 *[4] Let $X = R$ and suppose,*

$$d(x, y) = |2x - y|^2 + |2x + y|^2$$

then (X, d) is dqb −metric space with coefficient $k = 2$ but (X, d) is not a quasi-b −metric space. Also (X, d) is not dislocated quasi metric space.

Definition 1.4 *[4] A sequence $\{x_n\}$ in a dqb −metric space (X, d), dqb −converges to $x \in X$ if*

$$\lim_{n\to\infty} d(x_n, x) = 0 = \lim_{n\to\infty} d(x, x_n).$$

In this case x is called dqb –limit of $\{x_n\}$ and $\{x_n\}$ is said to be dqb –convergent to x, written as $x_n \to x$.

Definition 1.5 *A sequence $\{x_n\}$ in a dqb –metric space (X, d) is called as dqb –Cauchy sequence if*

$$\lim_{n,m\to\infty} d(x_n, x_m) = 0 = \lim_{n,m\to\infty} d(x_m, x_n).$$

Definition 1.6 *[4] A dqb –metric space (X, d) is said to be dqb –complete if every dqb –Cauchy sequence in it is dqb –convergent in X.*

Definition 1.7 *[2] A subset M of X is said to be closed if and only if for each sequence $\{x_n\}$ in X which converges to an element x, we have $x \in M$.*

Definition 1.8 *[2] A subset S of X is bounded if there exists $M \in (0, \infty)$ such that $d(x, y) \le M$ for all $x, y \in S$.*

Proposition 1.9 *[5] Every subsequence of a dqb –convergent sequence in a dqb –metric space (X, d) is dqb –convergent sequence.*

Proposition 1.10 *[5] Every subsequence of a dqb –Cauchy sequence in a dqb –metric space (X, d) is dqb –Cauchy sequence.*

Proposition 1.11 *[5]If (X, d) is a dqb –metric space then a function $f: X \to X$ is continuous if and only if $x_n \to x \Rightarrow fx_n \to fx$.*

Lemma 1.12 *[5] Limit of a dqb –convergent sequence in dqb –metric space is unique.*

Proposition 1.13 *If u is limit of some dqb –convergent sequence in a dqb –metric space (X, d) then $d(u, u) = 0$.*

Proof. $0 = \lim_{n\to\infty} d(x_n, u) = \lim_{n\to\infty} d(x_n, x_m) = d(u, u), m > n$.

Definition 1.14 *[14] An element $(x, y) \in X \times X$ is called coupled fixed point of the mapping $T: X \times X \to X$ if $T(x, y) = x$ and $T(y, x) = y$.*

Definition 1.15 *An element $(x, y) \in X \times X$ is called coupled coincidence point of the mapping $T : X \times X \to X$ and $h : X \to X$ if $T(x, y) = hx$ and $T(y, x) = hy$.*

Definition 1.16 *Suppose X is a non-empty set. Then the mappings $T: X \times X \to X$ and $h: X \to X$ are commutative if $hT(x, y) = T(hx, hy)$ for all $x, y \in X$.*

Definition 1.17 *[3] An element $(x, y, z) \in X \times X \times X$ is called a tripled fixed point of a mapping $F: X \times X \to X \to X$ if $x = F(x, y, z), y = F(y, z, x)$ and $z = F(z, x, y)$.*

2. Main Results

Theorem 2.1 *Let (X,d) be a complete dqb − metric space with coefficient k. Let $T : X \times X \times X \to X$ be continuous mapping such that*

$$d(T(x,y,z),T(u,v,w)) \le \alpha d(x,u) + \beta d(y,v) + \gamma d(z,w), \tag{2.1}$$

for all $x,y,z,u,v,w \in X$. Where $\alpha,\beta,\gamma \in [0,1)$ such that $0 < \alpha+\beta+\gamma < \frac{1}{k}$. Then there exists a tripled fixed point of T in X.

Proof. Let (x_0,y_0,z_0) be any arbitrarily fixed point in $X \times X \times X$. We set $T(x_0,y_0,z_0) = x_1$, $T(y_0,z_0,x_0) = y_1$ and $T(z_0,x_0,y_0) = z_1$.Thus we can construct three sequences of points in X namely $\{x_n\}$, $\{y_n\}$ and $\{z_n\}$, with $x_{n+1} = T(x_n,y_n,z_n)$, $y_{n+1} = T(y_n,z_n,x_n)$ and $z_{n+1} = T(z_n,x_n,y_n)$. Consider,

$$\begin{aligned} d(x_1,x_2) &= d(T(x_0,y_0,z_0),T(x_1,y_1,z_1)) \\ &\le \alpha d(x_0,x_1) + \beta d(y_0,y_1) + \gamma d(z_0,z_1) \end{aligned} \tag{2.2}$$

Similarly,

$$\begin{aligned} d(y_1,y_2) &= d(T(y_0,z_0,x_0),T(y_1,z_1,x_1)) \\ &\le \alpha d(y_0,y_1) + \beta d(z_0,z_1) + \gamma d(x_0,x_1) \end{aligned} \tag{2.3}$$

and

$$\begin{aligned} d(z_1,z_2) &= d(T(z_0,x_0,y_0),T(z_1,x_1,y_1)) \\ &\le \alpha d(z_0,z_1) + \beta d(x_0,x_1) + \gamma d(y_0,y_1). \end{aligned} \tag{2.4}$$

Now summing up (??),(??) and (??), we get,

$$d(x_1,x_2) + d(y_1,y_2) + d(z_1,z_2) \le (\alpha+\beta+\gamma)(d(x_0,x_1) + d(y_0,y_1) + d(z_0,z_1)). \tag{2.5}$$

Now consider,

$$\begin{aligned} d(x_2,x_3) &= d(T(x_1,y_1,z_1),T(x_2,y_2,z_2)) \\ &\le \alpha d(x_1,x_2) + \beta d(y_1,y_2) + \gamma d(z_1,z_2) \end{aligned} \tag{2.6}$$

Similarly,

$$\begin{aligned} d(y_2,y_3) &= d(T(y_1,z_1,x_1),T(y_2,z_2,x_2)) \\ &\le \alpha d(y_1,y_2) + \beta d(z_1,z_2) + \gamma d(x_1,x_2) \end{aligned} \tag{2.7}$$

and

$$\begin{aligned} d(z_2,z_3) &= d(T(z_1,x_1,y_1),T(z_2,x_2,y_2)) \\ &\le \alpha d(z_1,z_2) + \beta d(x_1,x_2) + \gamma d(y_1,y_2). \end{aligned} \tag{2.8}$$

Now summing up (??),(??) and (??), we get,

$$d(x_2,x_3) + d(y_2,y_3) + d(z_2,z_3) \le (\alpha+\beta+\gamma)(d(x_1,x_2) + d(y_1,y_2) + d(z_1,z_2)). \tag{2.9}$$

Using (2.5), we can write,

$$d(x_2,x_3) + d(y_2,y_3) + d(z_2,z_3) \le (\alpha+\beta+\gamma)^2(d(x_0,x_1) + d(y_0,y_1) + d(z_0,z_1)).$$

In general, for any $n \in N$, we get,

$$d(x_n,x_{n+1}) + d(y_n,y_{n+1}) + d(z_n,z_{n+1}) \le (\alpha+\beta+\gamma)^n(d(x_0,x_1) + d(y_0,y_1) + d(z_0,z_1)). \tag{2.10}$$

Thus writing, $d_n = d(x_n,x_{n+1}) + d(y_n,y_{n+1}) + d(z_n,z_{n+1})$ and $\alpha+\beta+\gamma = \delta$ we get

$$0 \le d_n \le \delta^n d_0. \tag{2.11}$$

With similar arguments we can prove that,

$$0 \le d'_n \le \delta^n d'_0 \tag{2.12}$$

where $d'_n = d(x_{n+1},x_n) + d(y_{n+1},y_n) + d(z_{n+1},z_n)$ and $d'_0 = (d(x_1,x_0) + d(y_1,y_0) + d(z_1,z_0))$. Now, for $m,n \in N$ and $n < m$, using triangular inequality of the definition,

$$d(x_n,x_m) \le kd(x_n,x_{n+1}) + k^2 d(x_{n+1},x_{n+2}) + \ldots + k^{m-n} d(x_{m-1},x_m)$$

Similarly,

$$d(y_n,y_m) \le kd(y_n,y_{n+1}) + k^2 d(y_{n+1},y_{n+2}) + \ldots + k^{m-n} d(y_{m-1},y_m)$$

and

$$d(z_n, z_m) \le kd(z_n, z_{n+1}) + k^2 d(z_{n+1}, z_{n+2}) + \ldots + k^{m-n} d(z_{m-1}, z_m)$$

Adding these inequalities and using equation (2.25)

$$d(x_n, x_m) + d(y_n, y_m) + d(z_n, z_m) \le k[d(x_n, x_{n+1}) + d(y_n, y_{n+1}) + d(z_n, z_{n+1})] + k^2[d(x_{n+1}, x_{n+2})$$
$$+d(y_{n+1}, y_{n+2}) + d(z_{n+1}, z_{n+2})] + \cdots + k^{m-n}[d(x_{m-1}, x_m)$$
$$+d(y_{m-1}, y_m) + d(z_{m-1}, z_m)]$$
$$\le [k\delta^n + k^2\delta^{n+1} + \ldots + k^{m-n}\delta^{m-1}]d_0$$
$$= [(k\delta)\delta^{n-1} + (k\delta)^2\delta^{n-1} + \cdots + (k\delta)^{m-n}\delta^{n-1}]d_0$$
$$\le \gamma^{n-1}\eta, \quad where \quad \eta \ge (m-n)d_0.$$

Taking limit as $n, m \to \infty$ we get

$$\lim_{m,n\to\infty} d(x_n, x_m) = 0, \ \lim_{m,n\to\infty} d(y_n, y_m) = 0 \ \ and \ \lim_{m,n\to\infty} d(z_n, z_m). \quad (2.13)$$

Similarly, using (2.26) and triangular inequality, we can prove that,

$$\lim_{m,n\to\infty} d(x_m, x_n) = 0\,, \ \lim_{m,n\to\infty} d(y_m, y_n) = 0 \ and \ \lim_{m,n\to\infty} d(z_m, z_n) = 0. \quad (2.14)$$

Thus from equations (2.27) and (2.28), $\{x_n\}, \{y_n\}$ and $\{z_n\}$ are Cauchy sequences in X. Since X is complete dqb −metric space, there exist $u, v, w \in X$ such that $x_n \to u, y_n \to v$ and $z_n \to w$. We claim that (u, v, w) is a tripled fixed point of T. Since T is continuous, for $r > 0$, there exists $\omega > 0$ such that,

$$d(T(x,y,z), T(u,v,w)) < \frac{r}{3k} \ \ whenever \ \ d(x,u) + d(y,v) + d(z,w) < \omega$$

Using definition of limit, for $\xi = \min\{\frac{r}{3k}, \frac{\omega}{3k}\}$, we can find $N_x, N_y, N_z \in N$ such that

$$d(x_n, u) < \xi, d(y_n, v) < \xi, d(z_n, w) < \xi \ \ whenever \ \ n \ge N_0 = \max\{N_x, N_y, N_z\}.$$

Consider for any $n \ge N_0$,

$$d(T(u,v,w), u) \le k[d(t(u,v,w), x_{n+1}) + d(x_{n+1}, u)]$$
$$= kd(T(u,v,w), T(x_n, y_n, z_n)) + kd(x_{n+1}, u)$$
$$< r.$$

Thus $d(T(u,v,w), u) = 0$. Similarly $d(u, T(u,v,w)) = 0$. Thus we see that $T(u,v,w) = u$. On the similar lines we can prove that $T(v,w,u) = v, \ and \ T(w,u,v) = w$. And hence (u,v,w) is tripled fixed point of T in X.

Theorem 2.2 *Let (X, d) be a complete dqb − metric space with coefficient k. Let $T : X \times X \times X \to X$ be continuous mapping such that*

$$d(T(x,y,z), T(u,v,w)) \le \frac{\alpha}{6}[d(x, T(x,y,z)) + d(y, T(y,z,x)) + d(z, T(z,x,y)) \quad (2.15)$$
$$+d(u, T(u,v,w)) + d(v, T(v,w,u)) + d(w, T(w,u,v))],$$

for all $x, y, z, u, v, w \in X$. Where $\alpha \in [0,1)$. Then there exists a tripled fixed point of T in X.

Proof. Let (x_0, y_0, z_0) be any arbitrarily fixed point in $X \times X \times X$. We set $T(x_0, y_0, z_0) = x_1$, $T(y_0, z_0, x_0) = y_1$ and $T(z_0, x_0, y_0) = z_1$. Thus we can construct three sequences of points in X namely $\{x_n\}$, $\{y_n\}$ and $\{z_n\}$, with $x_{n+1} = T(x_n, y_n, z_n)$, $y_{n+1} = T(y_n, z_n, x_n)$ and $z_{n+1} = T(z_n, x_n, y_n)$. Consider,

$$d(x_1, x_2) = d(T(x_0, y_0, z_0), T(x_1, y_1, z_1))$$
$$\le \frac{\alpha}{6}[d(x_0, T(x_0, y_0, z_0)) + d(y_0, T(y_0, z_0, x_0)) + d(z_0, T(z_0, x_0, y_0)) \quad (2.16)$$
$$+d(x_1, T(x_1, y_1, z_1)) + d(y_1, T(y_1, z_1, x_1)) + d(z_1, T(z_1, x_1, y_1))]$$
$$= \frac{\alpha}{6}[d(x_0, x_1) + d(y_0, y_1) + d(z_0, z_1)$$

$$+d(x_1,x_2)+d(y_1,y_2)+d(z_1,z_2)].$$

Similarly,

$$\begin{aligned}d(y_1,y_2) &= d(T(y_0,z_0,x_0),T(y_1,z_1,x_1)) \quad (2.17)\\ &\le \tfrac{\alpha}{6}[d(y_0,T(y_0,z_0,x_0))+d(z_0,T(z_0,x_0,y_0))+d(x_0,T(x_0,y_0,z_0))\\ &\quad +d(y_1,T(y_1,z_1,x_1))+d(z_1,T(z_1,x_1,y_1))+d(x_1,T(x_1,y_1,z_1))]\\ &= \tfrac{\alpha}{6}[d(x_0,x_1)+d(y_0,y_1)+d(z_0,z_1)\\ &\quad +d(x_1,x_2)+d(y_1,y_2)+d(z_1,z_2)].\end{aligned}$$

and

$$\begin{aligned}d(z_1,z_2) &= d(T(z_0,x_0,y_0),T(z_1,x_1,y_1))\\ &\le \tfrac{\alpha}{6}[d(z_0,T(z_0,x_0,y_0))+d(x_0,T(x_0,y_0,z_0))+d(y_0,T(y_0,z_0,x_0)) \quad (2.18)\\ &\quad +d(z_1,T(z_1,x_1,y_1))+d(x_1,T(x_1,y_1,z_1))+d(y_1,T(y_1,z_1,x_1))]\\ &= \tfrac{\alpha}{6}[d(x_0,x_1)+d(y_0,y_1)+d(z_0,z_1)\\ &\quad +d(x_1,x_2)+d(y_1,y_2)+d(z_1,z_2)].\end{aligned}$$

Now summing up (??),(2.17) and (??), we get,

$$d(x_1,x_2)+d(y_1,y_2)+d(z_1,z_2) \le \tfrac{\alpha}{2}[d(x_0,x_1)+d(y_0,y_1)+d(z_0,z_1)]. \quad (2.19)$$

Now consider,

$$\begin{aligned}d(x_2,x_3) &= d(T(x_1,y_1,z_1),T(x_2,y_2,z_2))\\ &\le \tfrac{\alpha}{6}[d(x_1,T(x_1,y_1,z_1))+d(y_1,T(y_1,z_1,x_1))+d(z_1,T(z_1,x_1,y_1))\\ &\quad +d(x_2,T(x_2,y_2,z_2))+d(y_2,T(y_2,z_2,x_2))+d(z_2,T(z_2,x_2,y_2))]\\ &= \tfrac{\alpha}{6}[d(x_1,x_2)+d(y_1,y_2)+d(z_1,z_2) \quad (2.20)\\ &\quad +d(x_2,x_3)+d(y_2,y_3)+d(z_2,z_3)].\end{aligned}$$

Similarly,

$$\begin{aligned}d(y_2,y_3) &= d(T(y_1,z_{1,}x_0),T(y_1,z_1,x_1)) \quad (2.21)\\ &\le \tfrac{\alpha}{6}[d(y_0,T(y_0,z_0,x_0))+d(z_0,T(z_0,x_0,y_0))+d(x_0,T(x_0,y_0,z_0))\\ &\quad +d(y_1,T(y_1,z_1,x_1))+d(z_1,T(z_1,x_1,y_1))+d(x_1,T(x_1,y_1,z_1))]\\ &= \tfrac{\alpha}{6}[d(x_0,x_1)+d(y_0,y_1)+d(z_0,z_1)\\ &\quad +d(x_1,x_2)+d(y_1,y_2)+d(z_1,z_2)].\end{aligned}$$

and

$$\begin{aligned}d(z_2,z_3) &= d(T(z_1,x_1,y_1),T(z_2,x_2,y_2))\\ &\le \alpha d(z_1,z_2)+\beta d(x_1,x_2)+\gamma d(y_1,y_2). \quad (2.22)\end{aligned}$$

Now summing up (??),(??) and (??), we get,

$$d(x_2,x_3)+d(y_2,y_3)+d(z_2,z_3) \le (\alpha+\beta+\gamma)(d(x_1,x_2)+d(y_1,y_2)+d(z_1,z_2)). (2.23)$$

Using (2.5), we can write,

$$d(x_2,x_3)+d(y_2,y_3)+d(z_2,z_3) \le (\alpha+\beta+\gamma)^2(d(x_0,x_1)+d(y_0,y_1)+d(z_0,z_1)).$$

In general, for any $n \in N$, we get,

$$d(x_n,x_{n+1})+d(y_n,y_{n+1})+d(z_n,z_{n+1}) \le (\alpha+\beta+\gamma)^n(d(x_0,x_1)+d(y_0,y_1)+d(z_0,z_1)). \quad (2.24)$$

Thus writing, $d_n = d(x_n,x_{n+1})+d(y_n,y_{n+1})+d(z_n,z_{n+1})$ and $\alpha+\beta+\gamma=\delta$ we get

$$0 \le d_n \le \delta^n d_0. \quad (2.25)$$

With similar arguments we can prove that,

$$0 \le d'_n \le \delta^n d'_0 \quad (2.26)$$

where $d'_n = d(x_{n+1},x_n)+d(y_{n+1},y_n)+d(z_{n+1},z_n)$ and $d'_0 = (d(x_1,x_0)+d(y_1,y_0)+d(z_1,z_0))$. Now, for $m,n \in N$ and $n < m$, using triangular inequality of the definition,

$$d(x_n,x_m) \le k d(x_n,x_{n+1}) + k^2 d(x_{n+1},x_{n+2}) + \ldots + k^{m-n} d(x_{m-1},x_m)$$

Similarly,

$$d(y_n, y_m) \le kd(y_n, y_{n+1}) + k^2 d(y_{n+1}, y_{n+2}) + \ldots + k^{m-n} d(y_{m-1}, y_m)$$

and

$$d(z_n, z_m) \le kd(z_n, z_{n+1}) + k^2 d(z_{n+1}, z_{n+2}) + \ldots + k^{m-n} d(z_{m-1}, z_m)$$

Adding these inequalities and using equation (2.25)

$$\begin{aligned} d(x_n, x_m) + d(y_n, y_m) + d(z_n, z_m) &\le k[d(x_n, x_{n+1}) + d(y_n, y_{n+1}) + d(z_n, z_{n+1})] + k^2[d(x_{n+1}, x_{n+2}) \\ &\quad + d(y_{n+1}, y_{n+2}) + d(z_{n+1}, z_{n+2})] + \cdots + k^{m-n}[d(x_{m-1}, x_m) \\ &\quad + d(y_{m-1}, y_m) + d(z_{m-1}, z_m)] \\ &\le [k\delta^n + k^2\delta^{n+1} + \ldots + k^{m-n}\delta^{m-1}]d_0 \\ &= [(k\delta)\delta^{n-1} + (k\delta)^2\delta^{n-1} + \cdots + (k\delta)^{m-n}\delta^{n-1}]d_0 \\ &\le \gamma^{n-1}\eta, \quad where \quad \eta \ge (m-n)d_0. \end{aligned}$$

Taking limit as $n, m \to \infty$ we get

$$\lim_{m,n\to\infty} d(x_n, x_m) = 0, \lim_{m,n\to\infty} d(y_n, y_m) = 0 \ and \ \lim_{m,n\to\infty} d(z_n, z_m). \quad (2.27)$$

Similarly, using (2.26) and triangular inequality, we can prove that,

$$\lim_{m,n\to\infty} d(x_m, x_n) = 0 \, , \lim_{m,n\to\infty} d(y_m, y_n) = 0 \ and \ \lim_{m,n\to\infty} d(z_m, z_n) = 0. \quad (2.28)$$

Thus from equations (2.27) and (2.28), $\{x_n\}, \{y_n\}$ and $\{z_n\}$ are Cauchy sequences in X. Since X is complete dqb −metric space, there exist $u, v, w \in X$ such that $x_n \to u, y_n \to v$ and $z_n \to w$. We claim that (u, v, w) is a tripled fixed point of T. Since T is continuous, for $r > 0$, there exists $\omega > 0$ such that,

$$d(T(x, y, z), T(u, v, w)) < \frac{r}{3k} \ \ whenever \ \ d(x, u) + d(y, v) + d(z, w) < \omega$$

Using definition of limit, for $\xi = \min\{\frac{r}{3k}, \frac{\omega}{3k}\}$, we can find $N_x, N_y, N_z \in N$ such that

$$d(x_n, u) < \xi, d(y_n, v) < \xi, d(z_n, w) < \xi \ \ whenever \ \ n \ge N_0 = \max\{N_x, N_y, N_z\}.$$

Consider for any $n \ge N_0$,

$$\begin{aligned} d(T(u, v, w), u) &\le k[d(t(u, v, w), x_{n+1}) + d(x_{n+1}, u)] \\ &= kd(T(u, v, w), T(x_n, y_n, z_n)) + kd(x_{n+1}, u) \\ &< r. \end{aligned}$$

Thus $d(T(u, v, w), u) = 0$. Similarly $d(u, T(u, v, w)) = 0$. Thus we see that $T(u, v, w) = u$. On the similar lines we can prove that $T(v, w, u) = v, \ and \ T(w, u, v) = w$. And hence (u, v, w) is tripled fixed point of T in X.

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