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On Fixed Points of Generalized Chatterjea Mappings in Parametric b-Metric Spaces

Dr. Pradip Golhare
Department of Mathematics
Sant Dnyaneshwar Mahavidyalaya, Soegaon
Maharashtra, India
Email: golhare@gmail.com

Abstract

In this paper, researcher introduce generalized Chatterjea mapping in parametric b-metric spaces. It has been proved a fixed point theorem for generalized Chatterjea type contraction in parametric b-metric spaces. It also included some supporting examples to validate the given theorem.

Keywords: Fixed point, parametric b-metric space, Banach contraction principle, Kannan contraction, Chatterjea contraction.Mathematics Subject Classification (2020): 47H10, 54H25.

1 Introduction

Fixed point theory has a very fundamental position in various branches of applied mathematics. One of the celebrated results of this theory is the Banach contraction principle, established by Banach [1] in 1922. Following this, many works have appeared in the literature of fixed point theory, which have proved many extensions and generalizations of the Banach contraction principle.

Several generalized metric structures have been introduced to extend the Banach contraction principle. One such structure is the b-metric space. Czerwik [4] proposed the concept of a b-metric space. The concept of a b-metric space arises by the replacement of the triangle inequality with a relaxed inequality that involves a constant coefficient b. In this way, the introduction of b-metric spaces has opened new horizons for research in the fixed point theory. More recently, Hussain et al. [7] introduced parametric b-metric spaces, which provide a broader framework for studying fixed point theorems for mappings involving parameter-dependent distance functions. Several fixed point results have been established in these spaces under various types of contractive conditions.

Various types of contractions have also been introduced in the literature of fixed point theory, like Kannan [9] contraction, Chatterjea [2] contraction etc. Petrov [14] introduced the concept of generalized Kannan contraction in metric spaces. And Păcurar et al. [13] introduced the concept of generalized Chatterjea contraction mapping in metric spaces, with a fixed point theorem for such mappings. Motivated by these developments, in this paper, we establish a fixed point theorem for generalized Chatterjea contraction mapping in complete parametric b-metric spaces. An example is included to illustrate the applicability of the obtained results.

2 Preliminaries

The notion of parametric b-metric spaces was introduced by Hussain et. al [7] as a generalization of b-metric spaces. In this section, we recall several definitions and basic results concerning parametric b-metric spaces.

Definition 2.1. [7] Let X be a nonempty set and let

$$d: X \times X \times [0,\infty) \to [0,\infty)$$

be a function. Then (X, d) is called a parametric b-metric space if there exists a constant $s \geq 1$ such that for all $x, y, z \in X$ and $t \geq 0$;

(i) $d(x, y, t) = 0$ if and only if $x = y$,

(ii) $d(x,y,t) = d(y,x,t)$,
(iii) $d(x,z,t) \leq s[d(x,y,t) + d(y,z,t)]$.
The constant s is called the coefficient of the parametric b-metric.
Definition 2.2. [7] Let (X,d) be a parametric b-metric space. A sequence $\{x_n\}$ in X is said to converge to $x \in X$ if
$$\lim_{n\to\infty} d(x_n,x,t) = 0$$
for every $t \geq 0$. In this case, we write $x_n \to x$.
Definition 2.3. [7] A sequence $\{x_n\}$ in a parametric b-metric space (X,d) is called a Cauchy sequence if
$$\lim_{m,n\to\infty} d(x_m,x_n,t) = 0$$
for every $t \geq 0$.
Definition 2.4. [7] A parametric b-metric space (X,d) is said to be complete if every Cauchy sequence in X converges to a point of X.
Definition 2.5. [7] Let (X,d) be a parametric b-metric space and $T: X \to X$ be a mapping. We say T is continuous at x in X, if for any sequence $\{x_n\}$ in X such that, $x_n \to x$ as $n \to \infty$ then, $Tx_n \to Tx$ as $n \to \infty$.
Definition 2.6. [13] Let (X,d) be a metric space with $|X| \geq 3$. A mapping $T: X \to X$ is called a generalized Chatterjea type mapping on X if there exists a constant $\alpha \in \left[0,\frac{1}{2}\right)$ such that
$$\begin{aligned} &d(Tx,Ty) + d(Ty,Tz) + d(Tz,Tx) \\ &\leq \alpha[d(x,Ty) + d(y,Tx) + d(y,Tz) + d(z,Tx) + d(z,Ty) + d(x,Tz)] \end{aligned}$$
for all three pairwise distinct points $x,y,z \in X$.
Using this definition, we define generalized Chatterjea type mapping in parametric b-metric spaces as follows:
Definition 2.7. Let (X,d) be a parametric b-metric space with $s > 1$ and $|X| \geq 3$. A mapping $f: X \to X$ is called a generalized Chatterjea type mapping on X if there exists a constant $\alpha \in \left[0,\frac{1}{2s^3+s^2+1}\right)$ such that
$$\begin{aligned} &d(fx,fy,t) + d(fy,fz,t) + d(fz,fx,t) \\ &\leq \alpha[d(x,fy,t) + d(y,fx,t) + d(y,fz,t) + d(z,fx,t) + d(z,fy,t) + d(x,fz,t)] \end{aligned}$$
for all three pairwise distinct points $x,y,z \in X$.
Lemma 2.1. Let (X,d) be a parametric b-metric space. If a sequence $\{x_n\}$ converges to $x \in X$, then it is bounded, i.e., there exists $M > 0$ such that
$$d(x_n,x,t) \leq M$$
for all n and $t \geq 0$.
Proof. Since $x_n \to x$, for every $t \geq 0$ we have
$$\lim_{n\to\infty} d(x_n,x,t) = 0.$$
Hence the sequence $\{d(x_n,x,t)\}$ is bounded. Therefore there exists $M > 0$ such that $d(x_n,x,t) \leq M$ for all n. □
Proposition 2.2. Let (X,d) be a parametric b-metric space. Then the limits of sequences are unique.
Proof. Suppose $\{x_n\}$ converges to x and y. Then
$$d(x,y,t) \leq s[d(x,x_n,t) + d(x_n,y,t)].$$
Letting $n \to \infty$, we obtain
$$d(x,y,t) = 0.$$
Hence $x = y$. □
Example 2.3. Let $X = \mathbb{R}$ and define
$$d(x,y,t) = t|x-y|.$$

Then (X,d) is a parametric b-metric space with coefficient $s=1$. Indeed,

$$d(x,z,t)=t|x-z|\le t(|x-y|+|y-z|)=d(x,y,t)+d(y,z,t).$$

Lemma 2.4. If $s\ge 1$, and $\alpha\in\left[0,\frac{1}{2s^3+s^2+1}\right)$ then

$$\alpha<\frac{1}{1+s^2}\ \text{ and }\ \alpha\le\frac{1}{1+s+s^2}.$$

Proof. Since $2s^3\ge s$ for $s\ge 1$, we get $2s^3+s^2+1\ge s+s^2+1$. So

$$\frac{1}{2s^3+s^2+1}\le\frac{1}{1+s+s^2}.$$

Also $2s^3+s^2+1\ge 1+s^2$, so $\frac{1}{2s^3+s^2+1}\le\frac{1}{1+s^2}$. □

Lemma 2.5. Suppose $d(x_{n+1},x_{n+2},t)\le hd(x_{n-1},x_n,t)$ for all $n\ge 1$, with $h\in[0,1)$. Then for every $k\ge 0$,

$$d(x_{2k},x_{2k+1},t)\le h^k d(x_0,x_1,t),\ \text{ and }\ d(x_{2k+1},x_{2k+2},t)\le h^k d(x_1,x_2,t).$$

Proof. We prove this by induction on k. For case $k=0$,

$$d(x_0,x_1,t)\le d(x_0,x_1,t)\ \text{ and }\ d(x_1,x_2,t)\le d(x_1,x_2,t)$$

are obvious.

Let us assume that

$$d(x_{2k},x_{2k+1},t)\le h^k d(x_0,x_1,t)\ \text{ and }\ d(x_{2k+1},x_{2k+2},t)\le h^k d(x_1,x_2,t)$$

for some $k\ge 0$.

From the assumption of the statement with $n=2k+1$,

$$d(x_{2k+2},x_{2k+3},t)\le hd(x_{2k},x_{2k+1},t)\le h\cdot h^k d(x_0,x_1,t)=h^{k+1}d(x_0,x_1,t).$$

Applying the assumption in the statement of the Lemma, taking $n=2k+2$;

$$d(x_{2k+3},x_{2k+4},t)\le hd(x_{2k+1},x_{2k+2},t)\le h\cdot h^k d(x_1,x_2,t)=h^{k+1}d(x_1,x_2,t).$$

Hence, the proof is complete. □

Corollary 2.6. Let $M=\max\big(d(x_0,x_1,t),d(x_1,x_2,t)\big)$ and $\mu=\sqrt{h}\in[0,1)$. Then

$$d(x_n,x_{n+1},t)\le\frac{M}{\mu}\mu^n\ \text{ for all } n\ge 0.$$

Proof. For $n=2k$, by Lemma 2.5,

$$d(x_n,x_{n+1},t)\le h^k d(x_0,x_1,t)\le M\mu^{2k}=M\mu^n\le\frac{M}{\mu}\mu^n.$$

If $n=2k+1$, $d(x_n,x_{n+1},t)\le h^k d(x_1,x_2,t)\le M\mu^{2k}=M\mu^{n-1}=\frac{M}{\mu}\mu^n$. □

Lemma 2.7. If $d(x_n,x_{n+1},t)\le C\mu^n$ for all $n\ge 0$, with $C>0$ and $0\le s\mu<1$, then $\{x_n\}$ is a Cauchy sequence.

Proof. Fix $t>0$ and $n>m\ge 0$. Repeated application of the triangle inequality of the parametric b-metric space gives

$$\begin{aligned}
d(x_m,x_n,t)&\le s[d(x_m,x_{m+1},t)+d(x_{m+1},x_n,t)]\\
&\le sd(x_m,x_{m+1},t)+s^2[d(x_{m+1},x_{m+2},t)+d(x_{m+2},x_n,t)]\\
&\le sd(x_m,x_{m+1},t)+s^2d(x_{m+1},x_{m+2},t)+s^3d(x_{m+2},x_{m+3},t)+\cdots\\
&\quad +s^{n-m-1}d(x_{n-2},x_{n-1},t)+s^{n-m}d(x_{n-1},x_n,t)\\
&\le\sum_{j=m}^{n-1}s^{j-m+1}d\big(x_j,x_{j+1},t\big)\\
&\le\sum_{j=m}^{n-1}s^{j-m+1}C\mu^j\le sC\mu^m\sum_{i=0}^{n-m-1}(s\mu)^i\le\frac{sC\mu^m}{1-s\mu}
\end{aligned}$$

as $s\mu<1$. Since $\mu<1$, the right hand side goes to 0 as $m\to\infty$. Hence $\{x_n\}$ is a Cauchy sequence. □

Lemma 2.8. Let f be a generalized Chatterjea mapping on a parametric b-metric space X. If $\{x_n\}$ is a sequence of points in X such that $x_{n+1} = fx_n, \forall n \in \mathbb{N} \cup \{0\}$. Then there is at most one index $n \geq 0$ with $x_n = p$, and at most one index n with $x_{n+1} = p$.
Proof. Suppose $x_n = p = x_m$ for some $0 \leq n < m$. Since $x_{k+1} = fx_k$, implies that $x_{n+i} = x_{m+i}$ for every $i \geq 0$. Let $r = m - n > 0$, then the sequence $\{d(x_k, x_{k+1}, t)\}_{k\geq n}$ is periodic with period $r, d\left(x_{n+j}, x_{n+j+1}, t\right) = d\left(x_{n+j+r}, x_{n+j+1+r}, t\right)$ for all $j \geq 0$. For fixed $t > 0$ such that $d(x_n, x_{n+1}, t) > 0$. Thus $d\left(x_{n+jr}, x_{n+1+jr}, t\right) = d(x_n, x_{n+1}, t) > 0$ for every $j \geq 0$, contradiction to $d(x_n, x_{n+1}, t) \to 0$. Hence, no such $m > n$ exists, i.e., at most one index n satisfies $x_n = p$. The similar arguments applied to the sequence $(x_{n+1})_{n\geq 0}$ show that at most one index n satisfies $x_{n+1} = p$. □
Unlike Păcurar and Popescu [13], who worked in metric spaces, our theorem is established in complete parametric b-metric spaces

3 Main Result

Theorem 3.1. Let (X, d) be a complete parametric b-metric space with coefficient $s > 1$ and $|X| \geq 3$. Let the mapping $f: X \to X$ satisfy the following conditions:
(i) $f(fx) \neq x$ for all $x \in X$ such that $fx \neq x$
(ii) f is a generalized Chatterjea type mapping on X. Then, f has a fixed point. The number of fixed points is at most two.
Proof. Let $x_0 \in X$ be arbitrary and define the sequence

$$x_{n+1} = fx_n, n = 0,1,2, ...$$

If $x_n = fx_n$ for some $n \geq 0$, then x_n is a fixed point of T, and the proof is complete. Hence, we may assume that

$$x_n \neq fx_n, \text{ for all } n \geq 0.$$

Since $x_n = fx_{n-1}$, it follows that

$$x_n \neq x_{n-1}, n \geq 1.$$

Moreover, by the hypothesis that $f^2x \neq x$ whenever $fx \neq x$, we have

$$x_{n+1} = f(fx_{n-1}) \neq x_{n-1}, n \geq 1.$$

Also,

$$x_{n+1} = fx_n \neq x_n, n \geq 0.$$

Therefore, the points x_{n-1}, x_n, and x_{n+1} are pairwise distinct for every $n \geq 1$. Taking

$$x = x_{n-1}, y = x_n, z = x_{n+1},$$

in (2.1), we obtain

$$\begin{aligned}&d(fx_{n-1}, fx_n, t) + d(fx_n, fx_{n+1}, t) + d(fx_{n+1}, fx_{n-1}, t)\\ &\leq \alpha[d(x_{n-1}, fx_n, t) + d(x_n, fx_{n-1}, t) + d(x_n, fx_{n+1}, t)\\ &+d(x_{n+1}, fx_{n-1}, t) + d(x_{n+1}, fx_n, t) + d(x_{n-1}, fx_{n+1}, t)]\end{aligned}$$

Hence,

$$\begin{aligned}&d(x_n, x_{n+1}, t) + d(x_{n+1}, x_{n+2}, t) + d(x_n, x_{n+2}, t)\\ &\leq \alpha[d(x_{n-1}, x_{n+1}, t) + d(x_n, x_{n+2}, t) + d(x_n, x_{n+1}, t) + d(x_{n-1}, x_{n+2}, t)]\end{aligned}$$

Since,

$$d(x_n, x_{n+2}, t) + d(x_n, x_{n+1}, t) \leq d(x_n, x_{n+1}, t) + d(x_{n+1}, x_{n+2}, t) + d(x_n, x_{n+2}, t),$$

we obtain

$$\begin{aligned}&(1-\alpha)[d(x_n, x_{n+1}, t) + d(x_{n+1}, x_{n+2}, t) + d(x_n, x_{n+2}, t)] \quad (3.1)\\ &\leq \alpha[d(x_{n-1}, x_{n+1}, t) + d(x_{n-1}, x_{n+2}, t)]. \quad (3.1)\end{aligned}$$

Using the parametric b-metric inequality, we have

$$\begin{aligned}d(x_{n-1}, x_{n+2}, t) &\leq s[d(x_{n-1}, x_n, t) + d(x_n, x_{n+2}, t)]\\ &\leq s\left[d(x_{n-1}, x_n, t) + s\left(d(x_n, x_{n+1}, t) + d(x_{n+1}, x_{n+2}, t)\right)\right]\\ &= sd(x_{n-1}, x_n, t) + s^2 d(x_n, x_{n+1}, t) + s^2 d(x_{n+1}, x_{n+2}, t).\end{aligned}$$

Substituting this in (3.1), we obtain

$$(1-\alpha)[d(x_n,x_{n+1},t)+d(x_n,x_{n+2},t)+d(x_{n+1},x_{n+2},t)]$$
$$\leq \alpha[d(x_{n-1},x_{n+1},t)+sd(x_{n-1},x_n,t)+s^2d(x_n,x_{n+1},t)+s^2d(x_{n+1},x_{n+2},t)]$$

Hence,

$$[(1-\alpha)-\alpha s^2][d(x_n,x_{n+1},t)+d(x_{n+1},x_{n+2},t)]+(1-\alpha)d(x_n,x_{n+2},t)$$
$$\leq \alpha[d(x_{n-1},x_{n+1},t)+sd(x_{n-1},x_n,t)].$$

Let $t>0$ be arbitrarily fixed. Let us write $D_n=d(x_n,x_{n+1},t)$ and $E_n=d(x_n,x_{n+2},t)$. The above inequality can be written as

$$[(1-\alpha)-\alpha s^2][D_n+D_{n+1}]+(1-\alpha)E_n \leq \alpha[E_{n-1}+sD_{n-1}]. \tag{3.2}$$

By the parametric b-metric triangle inequality,

$$E_{n-1}=d(x_{n-1},x_{n+1},t)\leq s[D_{n-1}+D_n].$$

Substituting into the right-hand side of inequality (3.2) we obtain,

$$[(1-\alpha)-\alpha s^2][D_n+D_{n+1}]+(1-\alpha)E_n \leq \alpha s[2D_{n-1}+D_n]. \tag{3.3}$$

Since $\alpha<1$, we have $(1-\alpha)E_n\geq 0$, so we can omit it from (3.3) to obtain,

$$[(1-\alpha)-\alpha s^2][D_n+D_{n+1}]\leq \alpha s[2D_{n-1}+D_n]. \tag{3.4}$$

Since, by Lemma 2.4, $\alpha<\frac{1}{1+s^2}\Longrightarrow(1-\alpha)-\alpha s^2>0=k$. If $(1-\alpha)-\alpha s^2=k$, then we can rewrite (3.4) as

$$kD_{n+1}\leq 2\alpha sD_{n-1}+(\alpha s-k)D_n. \tag{3.5}$$

Again using Lemma 2.4, $\alpha\leq\frac{1}{1+s+s^2}$, which implies that $\alpha s-k\leq 0$. Thus using $(\alpha s-k)D_n\leq 0$, (3.5) becomes

$$D_{n+1}\leq hD_{n-1},\ \text{with}\ h=\frac{2\alpha s}{k}. \tag{3.6}$$

Now to prove that the above sequence is Cauchy, consider

$$\alpha<\frac{1}{2s^3+s^2+1} \quad\Leftrightarrow \alpha(2s^3+s^2+1)<1$$
$$\Leftrightarrow 2\alpha s^3<(1-\alpha)-\alpha s^2$$
$$\Leftrightarrow s^2h<1$$

(3.7) From (3.7) $s^2h<1$, i.e. $s\mu<1$. Therefore, by Corollary 2.6, $(x_n,x_{n+1},t)\leq\frac{M}{\mu}\mu^n$.

Taking $C=\frac{M}{\mu}$ and using Lemma 2.7 we conclude that $\{x_n\}$ is Cauchy for every $t>0$. By completeness of parametric b-metric space (X,d), $\{x_n\}$ converges to some point $u\in X$. We now show that $fu=u$.

By Lemma 2.8, there is N_0 such that for all $n>N_0$;

$$x_n\neq u, x_{n+1}\neq u, x_n\neq x_{n+1}.$$

Hence $\{x_n,x_{n+1},u\}$ are pairwise distinct points for every $n>N_0$.

On the contrary, assume that $fu\neq u$. There exists $t_0>0$ such that

$$L=d(u,fu,t_0)>0.$$

Apply (2.1) with $x=x_n, y=x_{n+1}, z=u$ for $n>N_0$ to obtain

$$[(1-\alpha)-\alpha s^2]d(x_{n+2},fu,t_0)\leq \alpha[d(x_n,x_{n+2},t_0)+sd(x_n,x_{n+1},t_0)]$$
$$+\alpha sd(x_{n+1},fu,t_0) \tag{3.8}$$

Since $x_n\to u$, we have, for every fixed $t_0>0$,

$$\lim_{n\to\infty}d(x_n,x_{n+2},t_0)\leq\lim_{n\to\infty}s[d(x_n,u,t_0)+d(u,x_{n+2},t_0)]=0,$$
$$\lim_{n\to\infty}d(x_n,x_{n+1},t_0)=0$$

By the parametric b-metric triangle inequality,

$$d(x_{n+1},fu,t_0)\leq s[d(x_{n+1},u,t_0)+d(u,fu,t_0)]=sd(x_{n+1},u,t_0)+sL$$

so

$$\limsup_{n\to\infty}d(x_{n+1},fu,t_0)\leq sL$$

Also,

$$d(u, fu, t_0) \le s[d(u, x_{n+2}, t_0) + d(x_{n+2}, fu, t_0)],$$

so, taking $\limsup_{n\to\infty}$ and using $d(u, x_{n+2}, t_0) \to 0$,

$$L \le s \limsup_{n\to\infty} d(x_{n+2}, fu, t_0). \tag{3.9}$$

Now take $\limsup_{n\to\infty}$ in (3.8). We know that $k = (1-\alpha) - \alpha s^2 > 0$, so dividing by k,

$$\limsup_{n\to\infty} d(x_{n+2}, fu, t_0) \le \frac{\alpha}{k}[0 + s \cdot 0] + \frac{\alpha s}{k} \cdot sL = \frac{\alpha s^2}{k} L$$

Combining with (3.9),

$$L \le s \cdot \frac{\alpha s^2}{k} L = \frac{\alpha s^3}{k} L. \tag{3.10}$$

We claim that $\frac{\alpha s^3}{k} < 1$:

We know that $\alpha < \frac{1}{2s^3+s^2+1}$.

Since

$$s^3 + s^2 + 1 < 2s^3 + s^2 + 1 \Rightarrow \frac{1}{2s^3+s^2+1} < \frac{1}{s^3+s^2+1},$$

we get $\alpha < \frac{1}{2s^3+s^2+1} < \frac{1}{s^3+s^2+1}$.

Also,

$$\frac{\alpha s^3}{k} < 1 \Leftrightarrow \alpha s^3 < (1-\alpha) - \alpha s^2$$

$$\Leftrightarrow \alpha < \frac{1}{s^3+s^2+1}$$

which is true. So we conclude that $\frac{\alpha s^3}{k} < 1$. Therefore from (3.10), we get

$$L \le \frac{\alpha s^3}{k} L < L$$

which is a contradiction to $L > 0$. Therefore, our assumption $fu \neq u$ was false, and we must have $fu = u$, i.e. u is a fixed point of f. □

4 Examples

We consider the parametric b-metric space with coefficient $s = 2^{p-1}$

$$X = \mathbb{R}, d(x, y, t) = \frac{|x-y|^p}{t} \ (t > 0),$$

which is complete. We take $t = 1$.

Also, from the theorem above $\alpha \in \left[0, \frac{1}{2s^3+s^2+1}\right)$.

Example 4.1. Let $p = 2$, so $s = 2$ and $\alpha < \frac{1}{2(8)+4+1} = \frac{1}{21}$. Define $f: \mathbb{R} \to \mathbb{R}$ by

$$f(x) = 3 + \frac{1}{5}(x-3).$$

Note that $fx = x \Leftrightarrow \frac{4}{5}(x-3) = 0 \Leftrightarrow x = 3 \neq 0$. Unique fixed point $x_0 = 3$. From the construction of the sequence $x_{n+1} = fx_n$, write $y_n = x_n - 3$; then $y_{n+1} = \frac{1}{5} y_n$, so

$$D_n = d(x_n, x_{n+1}, 1) = |x_n - x_{n+1}|^2 = \left|y_n - \frac{1}{5} y_n\right|^2 = \left(\frac{4}{5}\right)^2 y_n^2 = \frac{16}{25} y_n^2.$$

Since $y_n = \left(\frac{1}{5}\right)^n y_0$, we get $y_n^2 = \left(\frac{1}{25}\right)^n y_0^2$, hence

$$D_n = \frac{16}{25} y_0^2 \cdot \left(\frac{1}{25}\right)^n.$$

In particular $D_{n+1} = \frac{1}{25} D_n \leq \frac{1}{25} D_{n-1}$ for all $n \geq 1$, so observe that

$$D_{n+1} \leq hD_{n-1}, h = \frac{1}{25}.$$

Now, to prove that the sequence defined as in the theorem is Cauchy, it is sufficient to verify that $s^2 h < 1$. Consider $s^2 h = 4 \cdot \frac{1}{25} = \frac{4}{25} < 1$, which shows that the sequence converges to the nonzero fixed point $x_0 = 3$.

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