TOTAL IRREGULARITY STRENGTH *OF A TOTAL GRAPH*

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ABSTRACT

An edge (A vertex) irregular total k-labeling of a graph G is such a labeling of the vertices and edges with integers 1, 2,..., k that the weights of any two different edges (vertices) are distinct, where the weight of an edge (a vertex) is the sum of the label of the edge (vertex) itself and the label of its incident vertices (edges). The minimum k, for which the graph G has an edge (a vertex) irregular total k-labeling, is called the total edge (vertex) irregularity strength of the graph G and is denoted by tes(G) (tvs(G)). In this paper, we determine the upper bound of the total edge (vertex) irregularity strength of the total graph of the star graph $K_{1,n}$, $n \geq 2$.

INTRODUCTION

Many practical problems in real life situations have motivated the study of labeling of a graph. Graph Labeling is a mathematical discipline of Graph Theory, closely related to the field of Computer Science. It concerns the assignment of values, usually represented by positive integers, to the edges and vertices of a graph. Graph labeling methods were motivated by application to Technology and Sports tournament Scheduling. By a labeling, we mean any mapping that carries a set of graph elements to set of numbers (usually positive integers) called labels. Labeled graphs are useful in many coding theory problems, ambiguity determination in x-ray crystallographic analysis, designing a communication network addressing system, Radio astrology problems etc.

We consider only finite, simple and undirected graphs, without loops and multiple edges. Let G = (V, E) be a graph with the vertex set V and the edge set E. The total graph T(G) of a graph G is the graph with vertex set $V \cup E$ and two vertices of T(G) are adjacent whenever they are neighbors in G. The star graph S_n is a complete bipartite graph $K_{1,n}$.

An edge irregular total k-labeling of a graph G is such a labeling of the vertices and edges with integers 1, 2,..., k that the weights of any two different edges are distinct, where the weight of an edge is the sum of the label of the edge itself and the labels of the two end vertices. The minimum k for which the graph G has an edge irregular total k-labeling, is called the total edge irregularity strength of the graph G and in denoted by tes(G). Similarly, a vertex irregular k-labeling of a graph G is such a labeling of the vertices and edges with integers 1, 2,..., k that the weights of any two different vertices are distinct, where the weight of a vertex is the sum of the label of the vertex itself and the labels of its incident edges. The minimum k, for which the graph G has a vertex irregular total k-labeling, is called the total vertex irregularity strength of G and is denoted by tvs(G).

The notions of the total edge irregularity strength and the total vertex irregularity strength were first introduced by Bača. et al

[1] in a recent paper. They are invariants analogous to irregularity strength of a graph G [3, 4, 8, 10, 11, 13, 14]. In [1], Bača et al. put forward the lower bounds of tes(G) and tvs(G) in terms of the maximum degree Δ, minimum degree δ, order $|E(G)|$ and size $|V(G)|$ of a graph G, which may be stated as in Theorem 1.1 and 1.2

Theorem 1.1: $\text{tes}(G) \geq \max\left\{\left\lceil \frac{\Delta+1}{2} \right\rceil, \left\lceil \frac{|E(G)|+2}{3} \right\rceil\right\}$

Theorem 1.2: $\text{tvs}(G) \geq \left\lceil \frac{|V(G)|+\delta}{\Delta+1} \right\rceil$

Based on these theorems, Bača et al [1] determined the exact values of the total edge irregularity strength of path P_n, star S_n, wheel W_n and friendship graph F_n and obtained the exact values of the total vertex irregularity strength of star S_n, complete graph K_n, cycle C_n and Prism D_n. Ivančo and Jendrol' [7] determined the total edge irregularity strength of any tree. Jendrol' et al [9] proved the exact values of the total edge irregularity strength of complete graphs and complete bipartite graphs. J. Miškuf and S. Jendrol' [12] determined the exact values of the total edge irregularity strength of mxn grids. Brandt et al [2] proved a conjecture about edge irregular total labeling. Tong Chunling et al [15] obtained the exact values of the total edge irregularity strength of some families of graphs including the generalized Petersen graph, Ladder, Mobius band etc. N. S. Hungund et al [5] determined total irregularity strength of Triangular Snake and Double Triangular Snake. N, S. Hungund [6] obtained total edge irregularity strength of generalized web graph $W_0(t, n)$. In this paper, we obtain the upper bound of the total edge (vertex) irregularity strength of the total graph of the star graph $K_{1,n}$, for $n \geq 2$.

MAIN RESULTS

In this section, we determine the upper bound of the total edge (vertex) irregularity strength of the total graph of the star graph $K_{1,n}$ $(n \geq 2)$.

Theorem 2.1: for $n \geq 2$,

$$\text{tes}(T(K_{1,n})) \leq \begin{cases} 2n, & \text{for } n = 2, 4 \\ 2n-1, & \text{for } n = 3 \\ 2n+1, & \text{for } n = 5 \\ \frac{n(n-1)}{2} + 1, & \text{for } n \text{ is even}, \; n \neq 2, 4 \\ \frac{n(n-1)}{2} & \text{for } n \text{ is odd}, \; n \neq 3, 5 \end{cases}$$

Proof: Let $G \cong T(K_{1,n})$, $n \geq 2$, be defined on $V = \{u, u_i, v_i : 1 \leq i \leq n\}$ and $E = \{uu_i, uv_i, u_iv_i, v_iv_{i+k} : 1 \leq i \leq n, k = 1, 2, \ldots, \left\lceil \frac{n-1}{2} \right\rceil\}$ where i is taken modulo n (replacing 0 by n) and $K_{1,n}$ is the star graph. Here G has $2n+1$ vertices and $\frac{n(n+5)}{2}$ edges.

A function f is defined on $V \cup E$ as follows

for $1 \leq i \leq n$, $f(u) = 1$

$f(u_i) = \left\lceil \frac{n}{2} \right\rceil$

$$f(v_i) = \begin{cases} 2n, & if\, n\; is\; even \\ 2n-1, & if\, n = 3 \\ 2n+1, & if\, n\; is\; odd,\, n \neq 3 \end{cases}$$

and $$f(uu_i) = \begin{cases} \frac{n}{2} + i, & if\; n\; is\; even \\ i, & if\; n = 3 \\ \left\lceil \frac{n}{2} \right\rceil + i, & if\;\; n\; is\;\; odd\,,\, n \neq 3 \end{cases}$$

$$f(uv_i) = i$$

$$f(u_i v_i) = \begin{cases} \frac{n}{2} + i + 1, & if\, n\; is\;\; even \\ \left\lceil \frac{n}{2} \right\rceil + i, & if\, n\; is\;\; odd \end{cases}$$

and $$f(v_i v_{i+k}) = \begin{cases} n(k-1) + i + 1, & if\, n\; is\; even \\ i + 2, & if\;\; n = 3 \\ n(k-1) + i, & if\;\; n\; is\;\; odd,\, n \neq 3 \end{cases}$$

where $k = 1, 2, \ldots, \left\lceil \frac{n-1}{2} \right\rceil$.

Since $wt\,(uv_i) = f(u) + f(uu_i) + f(u_i)$

$$= \begin{cases} n + 1 + i, & if\, n\; is\;\; even \\ \left\lceil \frac{n}{2} \right\rceil + 1 + i, & if\, n = 3 \\ 2\left\lceil \frac{n}{2} \right\rceil + 1 + i, & if\, n\; is\;\; odd,\;\; n \neq 3 \end{cases}$$

$wt\,(uv_i) = f(u) + f(uv_i) + f(v_i)$

$$= \begin{cases} 2n + 1 + i, & if\;\; n\;\; is\; even \\ 2n + i, & if\;\; n\;\; is\, odd \\ 2n + 2 + i, & if\, n\; is\;\; odd,\;\; n \neq 3 \end{cases}$$

$$wt\,(u_i v_i) = f((u_i) + f(u_i v_i) + f(v_i)$$

$$= \begin{cases} 3n - 1 + i, & if\ n\ is\ even \\ 2n - 1 + 2\left\lfloor \frac{n}{2} \right\rfloor + i, & if\ n = 3 \\ 2n + 1 + 2\left\lfloor \frac{n}{2} \right\rfloor + i, & if\ n\ is\ odd,\ n \neq 3 \end{cases}$$

$$wt\,(v_i v_{i+k}) = f(v_i) + f(v_i v_{i+k}) + f(v_{i+k})$$

$$= \begin{cases} n(k+3) + i + 1, & if\ \ n\ is\ even \\ 4n + i, & if\ \ n = 3 \\ n(k+3) + i + 2, & if\ \ n\ is\ odd,\ n \neq 3 \end{cases}$$

where $i + k$ subscripts modulo n,

the weights of the edges of $T(K_{1,n})$ under the labeling f are distinct and f is a mapping

from $V \cup E$ into $\{1,2,\ldots,2n\}$, if n = 2 and 4

$\{1,2,\ldots,2n-1\}$, if n = 3

$\{1,2,\ldots,2n+1\}$, if n = 5

$\{1,2,\ldots,\frac{n(n-1)}{2}+1\}$, if n is even, $n \neq 2,4$

and $\{1, 2,\ldots, \frac{n(n-1)}{2}\}$, if n is odd, $n \neq 3,5$.

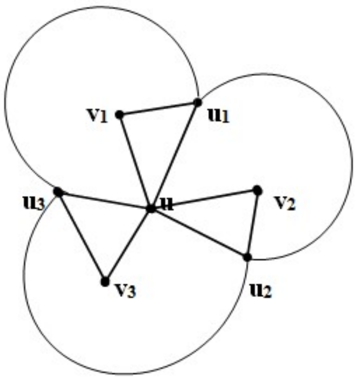

Fig 1: The total graph of $K_{1,3}$

Therefore total labeling of f has the required properties of all edge irregular total labeling. Hence, we have

$$tes(T(K_{1,n})) \leq \begin{cases} 2n, & for\ n = 2,4 \\ 2n - 1, & for\ n = 3 \\ 2n + 1, & for\ n = 5 \\ \dfrac{n(n-1)}{2} + 1, & for\ n\ is\ even,\ n \neq 2,4 \\ \dfrac{n(n-1)}{2} & for\ n\ is\ odd,\ n \neq 3,5 \end{cases}$$

Theorem 2.2: $tvs\ (T(K_{1,n})) \leq n,\ n \geq 2$.

Proof: Let $G \cong T\ (K_{1,n})$, for $n \geq 2$ be defined as in Theorem 2.1. Now consider a function f defined on $V \cup E$ as follows:

for $1 \leq i \leq n$, $f(u) = 1$

$f(u_i) = n-1, \quad n \geq 2$
$f(v_i) = 2$
and $f(uu_i) = 1$

$f(uv_i) = 1$
$f(u_iv_i) = i$
$f(v_iv_{i+k}) = 2.$

where $k = 1, 2, \ldots, \left\lceil \frac{n-1}{2} \right\rceil$ and i is taken modulo n.

Since $wt\ (u) = f(u) + \sum_{i=1}^{n} f(uu_i) + \sum_{i=1}^{n} f(uv_i)$

$= 1 + n + n$

$= 2n + 1$

$wt\ (u_i) = f(u_i) + f(uu_i) + f(u_iv_i)$

$= n + i$

and $wt\ (v_i) = f(v_i) + f(uv_i) + f(u_iv_i) + \sum f(v_iv_{i+k})$

$= 3 + i + \sum_{k=1}^{n-1} 2$

$= 3 + i + 2(n-1)$

$= 2n + i + 1$

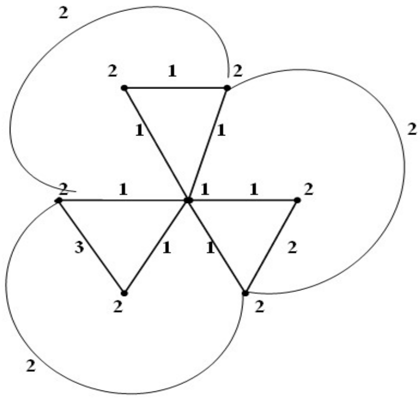

Fig 2: tvs (T($K_{1,3}$)) = 3

the weight of the vertices of T($K_{1,\,n}$) under the labeling f are distinct and f is a mapping from V∪E into {1, 2,..., n}. Clearly the total labeling f has required properties of a vertex irregular total labeling.

Therefore tvs (T($K_{1,\,n}$)) ≤ n, n ≥2.

Conjecture: Is total Irregularity strength of T($K_{1,\,n}$) = n ?

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